CDI 1 - TP **4o. MiniTeste** **LEE** **27/01/2022**

Justifique as suas respostas

ESBOÇO DE RESOLUÇÃO:

1. (**6 vals.**) Primitive a função: $\dfrac{x^4}{x^4-1}$

$$\frac{x^4}{x^4-1} = \frac{x^4-1+1}{x^4-1} = 1+\frac{1}{x^4-1} = 1+\frac{1}{(x^2-1)(x^2+1)} = 1+\frac{1}{(x-1)(x+1)(x^2+1)} =$$
$$= 1+\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1}$$

$$A = \left.\frac{1}{(x+1)(x^2+1)}\right|_{x=1} = \frac{1}{4} \qquad B = \left.\frac{1}{(x-1)(x^2+1)}\right|_{x=-11} = -\frac{1}{4}$$

$$\left(0 \underset{x\to+\infty}{\longleftarrow}\right) x\frac{1}{(x-1)(x+1)(x^2+1)} = x\frac{1/4}{x-1}+x\frac{-1/4}{x+1}+x\frac{Cx+D}{x^2+1}\left(\underset{x\to+\infty}{\longrightarrow} \frac{1}{4}-\frac{1}{4}+C+0\right)$$

donde $C=0$ e finalmente,

$$\left(-1=\right)\left.\frac{1}{(x-1)(x+1)(x^2+1)}\right|_{x=0} = \left.\frac{1/4}{x-1}\right|_{x=0}+\left.\frac{-1/4}{x+1}\right|_{x=0}+\left.\frac{D}{x^2+1}\right|_{x=0}\left(=-\frac{1}{4}-\frac{1}{4}+D\right)$$

donde $D=-1/2$. Portanto,

$$\begin{aligned}\mathbf{P}\frac{x^4}{x^4-1} &= \mathbf{P}1+\mathbf{P}\frac{1/4}{x-1}+\mathbf{P}\frac{-1/4}{x-1}+\mathbf{P}\frac{-1/2}{x^2+1} = \\ &= x+\frac{1}{4}\log|x-1|-\frac{1}{4}\log|x+1|-\frac{1}{2}\arctan x+c\end{aligned}$$

2. (**6 vals.**) Calcule $\displaystyle\lim_{x\to0}\frac{\int_0^x \sin(t^5)dt}{\int_0^{x^2}\sin(t^2)dt}$.

Seja $F(x)=\int_0^x \sin(t^5)dt$ e $G(y)=\int_0^y \sin(t^2)dt$. Então, pelo Teorema Fundamental da Análise, $G(x)$ e $F(x^2)$ são funções diferenciáveis já que $\sin(t^5)$ e $\sin(t^2)$ são funções contínuas. Por outro lado, $F(0)=0=G(0)$ donde $\displaystyle\lim_{x\to0}\frac{\int_0^x \sin(t^5)dt}{\int_0^{x^2}\sin(t^2)dt}=\frac{0}{0}$. Podemos então usar a regra de Cauchy:

$$\frac{\left(\int_0^x \sin(t^5)dt\right)'}{\left(\int_0^{x^2}\sin(t^2)dt\right)'} = \frac{\sin(x^5)}{\sin((x^2)^2)\cdot 2x} = \frac{1}{2}\cdot\frac{\sin(x^5)}{\sin(x^4)\cdot x} = \frac{1}{2}\cdot\frac{\sin(x^5)/x^5}{\sin(x^4)\cdot x/x^5} \underset{x\to0}{\longrightarrow} \frac{1}{2}\cdot\frac{1}{1} = \frac{1}{2}$$

Portanto, $\displaystyle\lim_{x\to0}\frac{\int_0^x \sin(t^5)dt}{\int_0^{x^2}\sin(t^2)dt}=\frac{1}{2}$

3. (**8 vals.**) Determine a natureza da série $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{\sqrt{n^4+n^2+1}}$.

$$\frac{n}{\sqrt{n^4+n^2+1}} \sim \frac{n}{n^2} = \frac{1}{n}.$$

Tem-se,

$$\frac{\frac{n}{\sqrt{n^4+n^2+1}}}{\frac{1}{n}} = \frac{n^2}{\sqrt{n^4+n^2+1}} = \frac{\sqrt{n^4}}{\sqrt{n^4+n^2+1}} = \sqrt{\frac{n^4}{n^4+n^2+1}} = \sqrt{\frac{1}{1+\frac{1}{n^2}+\frac{1}{n^4}}} \underset{n\to+\infty}{\longrightarrow} 1 \in \mathbf{R}^+$$

Então pelo critério do limite, as séries $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{\sqrt{n^4+n^2+1}}$ e $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n}$ têm a mesma natureza.

Como a série $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n}$ diverge (trata-se da série harmónica) então a série $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{\sqrt{n^4+n^2+1}}$ diverge.